Exercice 10.

Déterminez, si elle existe, la limite de la suite (u_n) proposée.

1. $u_n = \sqrt{2n^2 - 3}$
2. $u_n = \sqrt{4n^2 - 5} - 2n$

① $U_n = \sqrt{2n^2-3}$ $\quad \lim\limits_{n \to +\infty} \sqrt{2n^2-5} = +\infty$

car $\lim\limits_{n \to +\infty} 2n^2-5 = +\infty$

② $U_n = \sqrt{4n^2-5} - 2n$

$$\lim_{n \to +\infty} \sqrt{4n^2-5} - 2n = \lim_{n \to +\infty} \frac{(\sqrt{4n^2-5}-2n)(\sqrt{4n^2-5}+2n)}{\sqrt{4n^2-5}+2n}$$

$$= \lim_{n \to +\infty} \frac{4n^2-5-4n^2}{\sqrt{4n^2-5}+2n}$$

$$= \lim_{n \to +\infty} \frac{-5}{\sqrt{4n^2-5}+2n} = 0 \quad \text{car}$$

$\lim\limits_{n \to +\infty} \sqrt{4n^2-5}+2n = +\infty$

3)

$U_n = \sqrt{2n^2-5} - 2n$

$$= \lim_{n \to +\infty} n\left(\frac{\sqrt{2n^2-5}}{n} - \frac{2n}{n}\right)$$

$$= \lim_{n \to +\infty} n\left(\frac{\sqrt{2n^2-5}}{\sqrt{n^2}} - 2\right)$$

$\sqrt{n^2} = n$

$a > 0$
$b > 0$

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

$$= \lim_{n \to +\infty} n\left(\sqrt{\frac{2n^2-5}{n^2}} - 2\right) = \lim_{n \to +\infty} n\left(\sqrt{\frac{2n^2}{n^2} - \frac{5}{n^2}} - 2\right)$$

$$= \lim_{n \to +\infty} \underset{\downarrow\, +\infty}{n}\left(\underbrace{\sqrt{\frac{2}{1} - \frac{5}{n^2}} - 2}_{\sqrt{2}-2<0}\right)$$

$= -\infty$

4) $\lim\limits_{n \to +\infty} \sqrt{2n+1} \cdot \sqrt{2n-1} =$

$$= \lim_{n \to +\infty} \frac{(\sqrt{2n+1} - \sqrt{2n-1})(\sqrt{2n+1} + \sqrt{2n-1})}{\sqrt{2n+1} + \sqrt{2n-1}}$$

$$= \lim_{n \to +\infty} \frac{(\sqrt{2n+1}^2 - \sqrt{2n-1}^2)}{\sqrt{2n+1} + \sqrt{2n-1}}$$

$$= \lim_{n \to +\infty} \frac{(2n+1-(2n-1))}{\sqrt{2n+1} + \sqrt{2n-1}} = \lim_{n \to +\infty} \frac{2n+1-2n+1}{\sqrt{2n+1} + \sqrt{2n-1}}$$

$$= \lim_{n \to +\infty} \frac{2}{\underbrace{\sqrt{2n+1} + \sqrt{2n-1}}_{+\infty}} = 0$$

5) $$\lim_{n \to +\infty} \frac{n}{\sqrt{n+1} + \sqrt{n+2}} = \lim_{n \to +\infty} \frac{n}{\sqrt{n}\left(\frac{\sqrt{n+1}}{\sqrt{n}} + \frac{\sqrt{n+2}}{\sqrt{n}}\right)}$$

$$= \lim_{n \to +\infty} \frac{\sqrt{n}}{\sqrt{\frac{n+1}{n}} + \sqrt{\frac{n+2}{n}}} = \lim_{n \to +\infty} \frac{\sqrt{n}}{\sqrt{\frac{n}{n}+\frac{1}{n}} + \sqrt{\frac{n}{n}+\frac{2}{n}}}$$

$$= \lim_{n \to +\infty} \frac{\sqrt{n} \to +\infty}{\underbrace{\sqrt{1+\frac{1}{n}}}_{\downarrow\, 1} + \underbrace{\sqrt{1+\frac{2}{n}}}_{\downarrow\, 1}} = +\infty$$

$\boxed{\frac{n}{\sqrt{n}} = \sqrt{n}}$