Exemple 15.

Lever l'indétermination dans le cas $\infty - \infty$: Il faut factoriser par le terme de plus haut degré.

1. $\lim_{n \to +\infty} n^2 - 2n - 3 =$
2. $\lim_{n \to +\infty} -3n^3 + 3n^2 - n - 1 =$
3. $\lim_{n \to +\infty} 3n - \sqrt{n} =$

1) $\lim_{n \to +\infty} n^2 - 2n - 3 = \lim_{n \to +\infty} n^2\left(\frac{n^2}{n^2} - \frac{2n}{n^2} - \frac{3}{n^2}\right)$

$= \lim_{n \to +\infty} \underbrace{n^2}_{+\infty}\underbrace{\left(1 - \underbrace{\frac{2}{n}}_{0} - \underbrace{\frac{3}{n^2}}_{0}\right)}_{1} = +\infty$

Car $\lim_{n \to +\infty} \frac{2}{n} = \lim_{n \to +\infty} \frac{3}{n^2} = 0$

2) $\lim_{n \to +\infty} -3n^3 + 3n^2 - n - 1 = \lim_{n \to +\infty} \underbrace{n^3}_{+\infty}\left(\underbrace{-3}_{-3} + \underbrace{\frac{3}{n}}_{0} - \underbrace{\frac{1}{n^2}}_{0} - \underbrace{\frac{1}{n^3}}_{0}\right)$

$= -\infty$

$\lim_{n \to +\infty} -3n^3 + 3n^2 - n - 1 = \lim_{n \to +\infty} -3n^3\left(\frac{-3n^3}{-3n^3} + \frac{3n^2}{-3n^3} - \frac{n}{-3n^3} - \frac{1}{-3n^3}\right)$

$= \lim_{n \to +\infty} \underbrace{-3n^3}_{-\infty}\left(\underbrace{1 - \frac{1}{n} + \frac{1}{3n^2} + \frac{1}{3n^3}}_{1}\right) = -\infty$

3.

$\lim_{n \to +\infty} 3n - \sqrt{n} = \lim_{n \to +\infty} 3n\left(\frac{3n}{3n} - \frac{\sqrt{n}}{3n}\right) =$

$= \lim_{n \to +\infty} 3n\left(1 - \frac{\sqrt{n}}{3 \times \sqrt{n} \times \sqrt{n}}\right)$

$= \lim_{n \to +\infty} \underbrace{3n}_{+\infty}\left(1 - \underbrace{\frac{1}{3\sqrt{n}}}_{0}\right)$

$= +\infty$