

- 9. $u_n = \frac{2+5(-1)^n}{n}$
- 10. $u_n = \frac{10n-1}{n^2+1}$
- 11. $u_n = \frac{2n^2-1}{3n+7}$

9. $-1 < (-1)^n < 1$
 $-5 < 5(-1)^n < 5$
 $2-5 < 2+5(-1)^n < 7$

d'après le théorème
des gendarmes

$$\frac{-3}{n} < \frac{2+5(-1)^n}{n} < \frac{7}{n}$$

\downarrow \downarrow \downarrow
 0 0 0

$$\lim_{n \rightarrow +\infty} \frac{2+5(-1)^n}{n} = 0$$

10) $\lim_{n \rightarrow +\infty} \frac{10n-1}{n^2+1} = \lim_{n \rightarrow +\infty} n \left(\frac{10n-1}{n} \cdot \frac{1}{n} \right)$
 $n^2 \left(\frac{10}{n^2} + \frac{1}{n^2} \right)$

$$= \lim_{n \rightarrow +\infty} \frac{1}{n} \cdot \frac{10 - \frac{1}{n}}{1 + \frac{1}{n^2}} = 0$$

\downarrow \downarrow
 0 1

car $\lim_{n \rightarrow +\infty} \frac{1}{n} = 0$ et $\lim_{n \rightarrow +\infty} \frac{10 - \frac{1}{n}}{1 + \frac{1}{n^2}} = 10$
 et $\lim_{n \rightarrow +\infty} 1 + \frac{1}{n^2} = 1$

$$\lim_{n \rightarrow +\infty} \frac{10n-1}{n^2+1} = \lim_{n \rightarrow +\infty} \frac{10n \left(\frac{10n-1}{10n} \right)}{n^2 \left(\frac{n^2}{n^2} + \frac{1}{n^2} \right)} = \lim_{n \rightarrow +\infty} \frac{10}{n} \left(\frac{1 - \frac{1}{10n}}{1 + \frac{1}{n^2}} \right) = 0$$

11) $\lim_{n \rightarrow +\infty} \frac{2n^2-1}{3n+7} = \lim_{n \rightarrow +\infty} \frac{n^2 \left(2 - \frac{1}{n} \right)}{n \left(3 + \frac{7}{n} \right)}$
 $\lim_{n \rightarrow +\infty} \frac{n \left(2 - \frac{1}{n} \right)}{n \left(3 + \frac{7}{n} \right)} = +\infty$

car $\lim_{n \rightarrow +\infty} n = +\infty$; $\lim_{n \rightarrow +\infty} 2 - \frac{1}{n} = 2$ et $\lim_{n \rightarrow +\infty} 3 + \frac{7}{n} = 3$