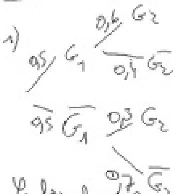
<u>EV022</u>

1)

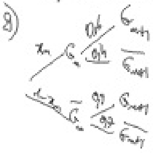


La formule des Proba totales montre que :

$P(G_2) = P(G_2 \cap G_1) + P(G_2 \cap \bar{G_1}) = P(G_1) P_{G_1}(G_2) + P(\bar{G_1}) P_{\bar{G_1}}(G_2) = 0{,}5 \times 0{,}6 + 0{,}5 \times 0{,}3 = 0{,}45$

$P(\bar{G_2}) = 1 - P(G_2) = 0{,}55$

2)



La formule des proba. totales montre que

$x_{n+1} = P(G_{n+1}) = P(G_{n+1} \cap G_n) + P(G_{n+1} \cap \bar{G_n})$

$= P(G_n) P_{G_n}(G_{n+1}) + P(\bar{G_n}) \times P_{\bar{G_n}}(G_{n+1})$

$= x_n \times 0{,}6 + y_n \times 0{,}3$

$= 0{,}6 x_n + 0{,}3 y_n$

$y_{n+1} = P(\overline{G_{n+1}}) = P(G_n \cap \overline{G_{n+1}}) + P(\bar{G_n} \cap \overline{G_{n+1}}) = P(G_n) P_{G_n}(\overline{G_{n+1}}) + P(\bar{G_n}) P_{\bar{G_n}}(\overline{G_{n+1}})$

$= x_n \times 0{,}4 + y_n \times 0{,}7 = 0{,}4 x_n + 0{,}7 y_n$.

3) a) Soit $n \in \mathbb{N}^*$, $v_{n+1} = x_{n+1} + y_{n+1} = 0{,}6 x_n + 0{,}3 y_n + 0{,}4 x_n + 0{,}7 y_n = x_n + y_n$

$= v_n$ donc v est constante.

Or $v_1 = x_1 + y_1 = 0{,}5 + 0{,}5 = 1$ donc v est constante égale à 1.

Ou $v_n = x_n + y_n = P(G_n) + P(\bar{G_n}) = 1$

3.b) $w_{n+1} = 4x_{n+1} - 3y_{n+1}$

$= 4(0{,}6x_n + 0{,}3y_n) - 3(0{,}4x_n + 0{,}7y_n)$

$= 2{,}4x_n + 1{,}2y_n - (1{,}2x_n + 2{,}1y_n)$

$= 2{,}4x_n - 1{,}2x_n + 1{,}2y_n - 2{,}1y_n$

$= 1{,}2x_n - 0{,}9y_n$

$= 0{,}3(4x_n - 3y_n) = 0{,}3\, w_n$

w est géo de raison $0{,}3$ et de 1er terme $w_1 = 4x_1 - 3y_1 = 4 \times 0{,}5 - 3 \times 0{,}5 = 0{,}5$

$w_n = 0{,}5 (0{,}3)^{n-1}$, $n \in \mathbb{N}$

4) a) On sait que $w_n = 4x_n - 3y_n$

Or $w_n = 0{,}5(0{,}3)^{n-1}$ et $x_n + y_n = 1$

c'est à dire

$y_n = 1 - x_n$

donc $w_n = 4x_n - 3y_n$

$0{,}5(0{,}3)^{n-1} = 4x_n - 3(1 - x_n)$

$\Leftrightarrow 0{,}5(0{,}3)^{n-1} = 4x_n - 3 + 3x_n$

$\Leftrightarrow -7x_n = -0{,}5(0{,}3)^{n-1} - 3$

$\Leftrightarrow x_n = \dfrac{-0{,}5(0{,}3)^{n-1} - 3}{-7}$

$\Leftrightarrow \dfrac{0{,}5(0{,}3)^{n-1} + 3}{7} = x_n$

4) b) Si $-1 < q < 1$ alors $\lim\limits_{n \to +\infty} q^n = 0$

Or $-1 < 0{,}3 < 1$ donc $\lim\limits_{n \to +\infty} (0{,}3)^{n-1} = 0$ *

donc $\lim\limits_{n \to +\infty} \dfrac{0{,}5(0{,}3)^{n-1} + 3}{7} = \dfrac{3}{7}$

* et $\lim\limits_{n \to +\infty} 0{,}5(0{,}3)^{n-1} = 0$

Laura a une chance de gagner de $\dfrac{3}{7}$